PENRITH HIGH SCHOOL

2015
HSC TRIAL EXAMINATION

Mathematics

General Instructions:

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Answer all Questions in the booklets provided

Total marks–100

SECTION I **Pages 3–6**

10 marks
- Attempt Questions 1–10
- Allow about 15 minutes for this section

SECTION II **Pages 7–13**

90 marks
- Attempt Questions 11–16
- Allow about 2 hours 45 minutes for this section

Student Name: ______________________ **Teacher Name:** ______________________

This paper MUST NOT be removed from the examination room

Assessor: Dr. Anju Katyal

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section
Use the provided multiple–choice answer sheet for Questions 1–10

1. The expression below correct to three significant figures is

$$\sqrt{\frac{6.25^2 + 12.125}{2.751 \times 2.11}}$$

(A) 2.96

(B) 2.97

(C) 2.90

(D) 2.969

2. Which of the following represents the domain of the function $f(x) = \sqrt{16 - x^2}$?

(A) $x \neq \pm 4$

(B) All real x values.

(C) $-4 \leq x \leq 4$

(D) $-4 < x < 4$

3. The 7th term of an arithmetic sequence is 11 and the 21st term is 53. The common difference is given by

(A) $d = -3$

(B) $d = 3$

(C) $d = 6$

(D) $d = -6$.

4. For what values of k will the geometric series $1 + 5k + 25k^2 + 125k^3 + ...$ have a limiting sum?

(A) $-1 \leq k \leq 1$

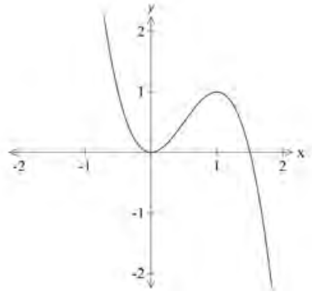
(B) $-\frac{1}{5} \leq k \leq \frac{1}{5}$

(C) $-\frac{1}{5} < k < \frac{1}{5}$

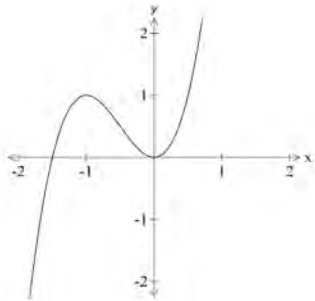
(D) $k < \frac{1}{5}$

5. Which of the following is the graph of $f(x) = 2x^3 - 3x^2$?

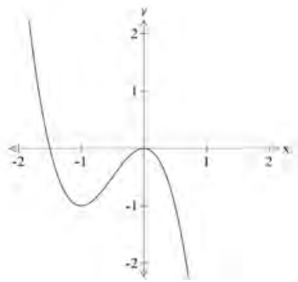
(A)



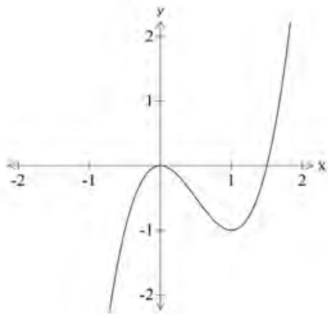
(B)



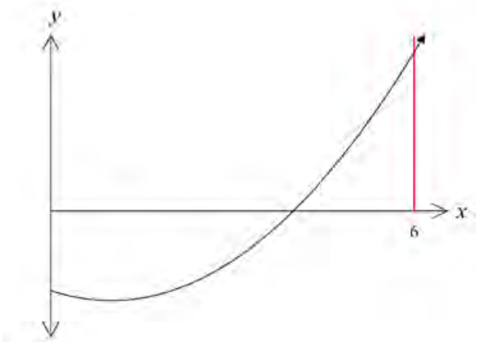
(C)



(D)



6. The diagram below shows the graph of $y = x^2 - 2x - 8$.



What is the correct expression for the area bounded by the *x*-axis and the curve $y = x^2 - 2x - 8$ between $0 \le x \le 6$?

(A) $A = \int_0^5 x^2 - 2x - 8dx + \left| \int_5^6 x^2 - 2x - 8dx \right|$

(B) $A = \int_0^4 x^2 - 2x - 8dx + \left| \int_4^6 x^2 - 2x - 8dx \right|$

(C) $A = \left| \int_0^5 x^2 - 2x - 8dx \right| + \int_5^6 x^2 - 2x - 8dx$

(D) $A = \left| \int_0^4 x^2 - 2x - 8dx \right| + \int_4^6 x^2 - 2x - 8dx$

7. A bag contains 11 balls of which 4 are blue and the rest are white. One ball is selected at random and removed from the bag. Another ball is selected and removed from the bag. What is the probability that both balls are white?

(A) $\dfrac{42}{110}$

(B) $\dfrac{49}{110}$

(C) $\dfrac{42}{122}$

(D) $\dfrac{49}{122}$

8. The solution to the equation $\log_e(x+2)-\log_e x=\log_e 4$ is given by

(A) $\dfrac{2}{5}$

(B) $\dfrac{2}{3}$

(C) $\dfrac{3}{2}$

(D) $\dfrac{5}{2}$

9. A particle is moving in a straight line with velocity $v=1-2e^{-3t}$.
Initially the particle is at the origin, t is measured in seconds and v in metres per second.

Which of the following statements is true?

(A) velocity is 2 m/s as t approaches infinity.

(B) velocity is 1 m/s as t approaches infinity.

(C) velocity is 3 m/s as t approaches infinity.

(D) The particle is at rest for larger values of t.

10. A particle moves so that at a time t seconds its position x metres is given by

$$x = 5 + \ln(2t + 1)$$

Which of the following statements is correct?

(A) The initial displacement is 5 m.

(B) The velocity after 2 seconds is 40 cm.

(C) The acceleration is always negative for any values of t.

(D) All of the above.

END OF SECTION I

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 11–16, your responses should include relevant mathematical reasoning and/or calculations

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Question 11 (15 marks) Use the Question 11 Writing Booklet.

a) Calculate correct to one decimal place the value of $\sqrt{\dfrac{2xy}{z}}$ when $x = 4.3$, $y = 6.7$ and $z = 4.9$. 1

b) Simplify the expression given by $\dfrac{x - x^{-1}}{1 + x^{-1}}$ 2

c) Differentiate with respect to x:

i. $11 + 2x^3$ 1
ii. $e^x \cos x$ 2
iii. $\log(x^2 + 1)$ 2

d) A parabola has an equation given by $y = \dfrac{1}{2}(x^2 - 4x + 5)$.

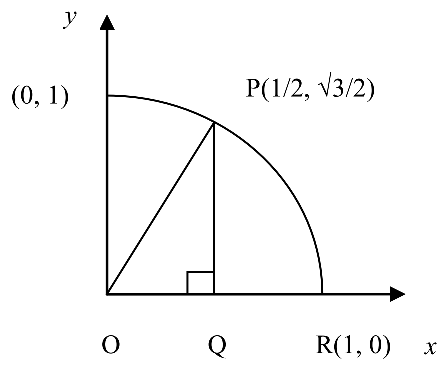
i. Express the above equation in the form $(y - q) = 4a(x - p)^2$. 2
ii. Find the co-ordinates of the vertex and focus of the parabola. 2
iii. Find the equation of the directrix of the parabola. 1
iv. Sketch the locus of P, indicating all the above information. 2

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

a) Convert $\frac{4\pi}{5}$ radians to degrees. 1

b) The first quadrant of the circle $x^2 + y^2 = 1$ is shown below. A Point, $P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ lies on the circle and the line PQ is perpendicular to the x-axis.

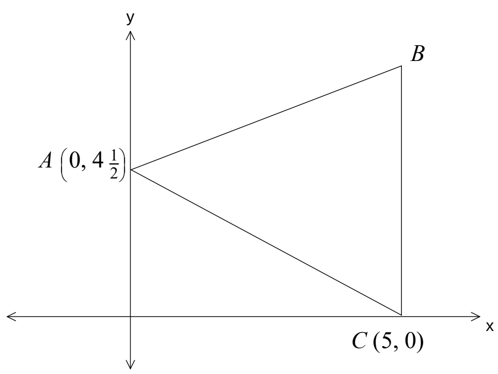


i. Show that the exact value of $\angle POQ = \frac{\pi}{3}$. 1

ii. Find the area of the sector POR and ΔPOQ 3

iii. Hence, find the exact shaded area. 1

c)



The lines AB and CB have equations $x - 2y + 9 = 0$ and $4x - y - 20 = 0$ respectively.

i. Show that the coordinates of the point B are given by (7, 8) 1

ii. Show that the equation of the line AC is $9x + 10y - 45 = 0$. 2

iii. Calculate the distance AC in exact form. 2

iv. Find the equation of the line perpendicular to BC which passes through A. 2

v. Calculate the shortest distance between the point B and the line AC. 1

vi. Hence find the area of the triangle ABC. 1

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

a) Find all values of θ such that $2\cos 2\theta = 1, 0 \le \theta \le 360^0$. 2

b) Evaluate $\lim\limits_{x \to 0} \dfrac{2\sin\frac{x}{2}}{x}$ 2

c) Prove that $\tan\theta(1-\cot^2\theta)+\cot\theta(1-\tan^2\theta)=0$ 2

d) Ashleigh plans to deposit a sum of money into an account which guarantees to pay her 1% interest each month on the balance of her account at the time. Immediately each interest payment is made, Ashleigh intends to withdraw \$500. She has no intention of ever adding to her initial deposit.

Using M to signify the initial deposit and A_n to represent the value of the investment after *n* withdrawals,

i. Write an expression for the value of her investment immediately after the first withdrawal. 1

ii. Show that when she has made the third withdrawal, the balance of her account will be $A_3 = \left(M(1.01)^3 - 500(1+1.01+1.01^2)\right)$ 2

iii. Write the expression for A_n. 1

iv. Ashleigh wants her deposit to be sufficient for her to be able to make withdrawals in this manner for 5 years. Show that her initial deposit needs to be \$22500 (to the nearest \$100). 2

e) If α and β are the roots of the equation $x^2-5x+2=0$.

Find, without solving, the values of:

i. $\alpha+\beta$ and $\alpha\beta$ 1

ii. $\left(\alpha^2+\dfrac{1}{\beta}\right)\left(\beta^2+\dfrac{1}{\alpha}\right)$ 2

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet.

a) Evaluate $\sum_{n=1}^{n=3} n^2(n+1)$ 1

b) Evaluate the expression $\log_2(mn)^3$ correct to two decimal places.
It is given that $\log_2 m = 0.2134$ and $\log_2 n = 0.3142$ 2

c) Consider the curve $y = -x^3 + 3x^2 + 9x - 11$.

i. Show that $\frac{dy}{dx} = -3(x-3)(x+1)$. 2

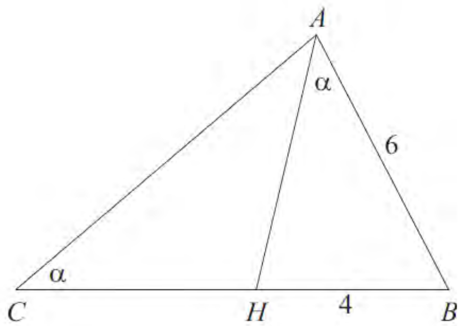
ii. Find the co-ordinates of any stationary points and show that there is one point of minima and one point of maxima. 3

iii. Find the co-ordinates of the points of inflexion, if any. 1

iv. Sketch the curve, clearly showing the y-intercept and all the stationary points and inflexion point(s). 2

v. For what values of x is the curve concave up? 1

d)



In the diagram above, $\angle BCA = \angle BAH = \alpha$, $AB = 6$ and $BH = 4$.

i. Show that $\Delta ABC \ ||| \ \Delta HBA$ 2

ii. Hence, or otherwise, find the length HC. 1

End of Question 14

Question 15 (15 marks) Use the Question 15 Writing Booklet

a) Find the exact value of the following definite integrals:

i. $\int_2^6 \frac{1}{x+2}dx$ 2

ii. $\int_0^1 (e^{2x}+1)dx$ 2

iii. $\int_{\pi/4}^{\pi/3} 3\sec^2(x)dx$ 2

b) A company decided to raise money for the end-of-season trip, 100 tickets were sold and two prizes were offered. Two tickets were drawn without replacement to determine the prize-winners.

Rocky bought some of the tickets. The probability that he won both prizes was $\frac{2}{275}$.

Find:

i. The number of tickets bought by Rocky. 1

ii. The probability of his winning at least one prize. 1

c) Consider the function given by the equation $y = \cos 2x$.

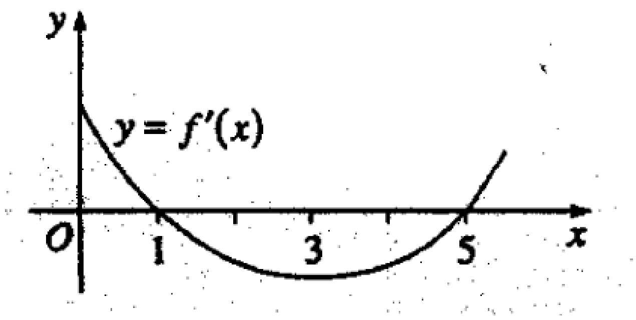
i. Sketch the graph of the function $y = \cos 2x$ for $-\pi \le x \le \pi$. 2

ii. On the same diagram, sketch the line $y = 1 - x$ 1

iii. Hence, determine the number of solutions of the equation $1 - x = \cos 2x$ 1

d) Use the Trapezoidal Rule with three values to estimate the area bounded by $y = \cos^2 x$, $x = 0$, $x = \frac{\pi}{2}$ and the x-axis to 3 decimal places. 2

e) Using the sketch of the gradient function drawn below, sketch its primitive function. 1



End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet.

a) For what value (s) of k does $3x^2 + 2x + k = 0$ have real roots? 1

b) Find the equation of the normal to the curve $y = 2\ln(x)$ at the point $x = e$. 2

c) A particle moves with velocity v m/s in the time t seconds according to:

$$v = \frac{5}{\sqrt{1+3t}}$$

Find

i. the acceleration as a function of time t. 2

ii. The displacement x as a function of time t if initially the particle was 2 metres to the right of the origin. 2

d) The population W of Williamtown is increasing exponentially according to the equation $W = W_0e^{0.02t}$ while the population H of Hectorville is decreasing exponentially according to the equation $H = H_0e^{-0.01t}$.

If the current populations of Williamtown and Hectorville are 8000 and 12000 respectively, how long, to the nearest year, will it be before their population are the same? 3

e) A box, open at the top, is to be made from cardboard. The base of the box is a square of side x cm and its height is y cm.

i. If the volume of the box is to be 32 cm^3, show that $y = \frac{32}{x^2}$. 1

ii. Show that the area of cardboard needed will be $A = x^2 + \frac{128}{x}$ cm^2. 2

iii. Find the dimensions of the box if this area is to be a minimum. 2

End of Question 16

End of paper

Exam 2unit Trial 2015	MATHEMATICS	: Question 11

Suggested Solutions	Marker's Comments

MC Answers

1B 2C 3B

4C 5D 6D

7A 8B 9B

10C

a) $\sqrt{\frac{2xy}{z}}$

$\sqrt{\frac{2(4\cdot 3)(6\cdot 7)}{4\cdot 9}}$

$= 3\cdot 4$ (correct to 1 d.p)

well done

b) $\frac{x - \frac{1}{x}}{1 + \frac{1}{x}} = \frac{\frac{x^2-1}{x}}{\frac{x+1}{x}}$

$= \frac{(x-1)(x+1)}{(\cancel{x+1})}$

$= x-1$

those students who didn't convert to $\frac{1}{x}$ got this wrong. Some tried to (rationalise) the denominator

c) (i) $6x^2$

(ii) $e^x \cos x = vu' + uv'$ ($u = e^x$, $v = \cos x$)

$= \cos x\, e^x + e^x(-\sin x)$

$= e^x(\cos x - \sin x)$

well done

well done

(iii) $\frac{2x}{x^2+1}$

d) $y = \frac{1}{2}(x^2 - 4x + 5)$

(i) $2y = (x-2)^2 + 1$

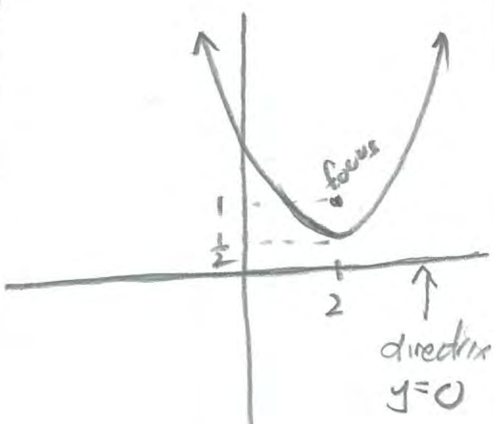
$2y - 1 = (x-2)^2$

$2(y - \frac{1}{2}) = (x-2)^2$

$= 4(\frac{1}{2})(y - \frac{1}{2}) = (x-2)^2$

(ii) vertex $(2, \frac{1}{2})$ focus $(2, 1)$

(iii) $(3, 0)$



It was obvious if students understood the concept and knew which direction to take in this question

Exam 2Unit Trial 2015 MATHEMATICS : Question 12

Suggested Solutions	Marker's Comments

a) $\dfrac{4(180)}{5} = 144^\circ$

well done.

b)

$\tan\theta = \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$\tan\theta = \sqrt{3}$

$\therefore \theta = \dfrac{\pi}{3}$

well done

(ii) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\cdot 1^2 \cdot \frac{\pi}{3}$

Area of sector $= \dfrac{\pi}{6}$

$A = \frac{1}{2}bh$

$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}$

Area of $\Delta POQ = \dfrac{\sqrt{3}}{8}$

those who knew formula got this correct

(iii) $\dfrac{\pi}{6} - \dfrac{\sqrt{3}}{8}$

$= \dfrac{8\pi - 6\sqrt{3}}{48}$

c(i) $x - 2y + 9 = 0$ — ①

$4x - y - 20 = 0$ — ②

①×4 $4x - 8y + 36 = 0$ — ③

② ③ $7y - 56 = 0$

$7y = 56$

$y = 8$

sub in ① $x - 2(8) + 9 = 0$

$x = 7$

$\therefore B(7, 8)$

some proved by showing (7,8) satisfies bothe equations

Exam	MATHEMATICS	: Question...12

Suggested Solutions	Marker's Comments
(ii) $M_{AC} = \frac{-4.5}{5}$ $= -\frac{9}{10}$ $y - y_1 = m(x - x_1)$ $y - 0 = \frac{-9}{10}(x-5)$ $10y = -9x + 45$ $9x + 10y - 45 = 0$	well done
(iii) $\sqrt{4.5^2 + 5^2}$ $= \sqrt{\frac{181}{4}}$ $= \frac{\sqrt{181}}{2}$	students who didn't leave in exact form had troubles with part (v)
(iv) $4x - y - 20 = 0$ $(0, 4.5)$ $M_{BC} = \frac{8}{2}$ $= 4$ $\perp M_{BC} = -\frac{1}{4}$ $y - 4.5 = -\frac{1}{4}(x - 0)$ $4y - 18 = -x$ $x + 4y - 18 = 0$	some students did not understand what they were finding
(v) $9x + 10y - 45 = 0$ $(7, 8)$ $d = \frac{\lvert Ax_1 + By_1 + C \rvert}{\sqrt{A^2 + B^2}}$ $= \frac{9(7) + 10(8) - 45}{\sqrt{9^2 + 10^2}} = \frac{98}{\sqrt{181}}$ (vi) Area $\triangle ABC$ $= \frac{1}{2} \times \frac{98}{\sqrt{181}} \times \frac{\sqrt{181}}{2}$ $= \frac{98}{4}$ $= 24.5.$	

Exam Mathematics Trial 2015 MATHEMATICS	: Question...... 13a,b,c,d
Suggested Solutions	Marker's Comments

a. $\lim_{x \to 0} \dfrac{\sin \frac{x}{2}}{\frac{x}{2}} = 1$

multiply numerator and denominator by 2

$\lim_{x \to 0} \dfrac{2\sin \frac{x}{2}}{x} = 1$

Many students didn't know the rule $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$

b. $2\cos 2\theta = 1$

$\cos 2\theta = \frac{1}{2}$

$2\theta = 60°, 300°, 420°, 660°, \ldots$

$\theta = 30°, 150°, 210°, 330°$

c. $\tan\theta(1-\cot^2\theta) + \cot\theta(1-\tan^2\theta) = 0$

Expand LHS

LHS $= \tan\theta - \cot\theta + \cot\theta - \tan\theta$

$= 0$

$=$ RHS

Best method: expand the brackets

d. $\dfrac{d^2y}{dx^2} = 3$

$\dfrac{dy}{dx} = 3x + C$

$x = 1$ $\dfrac{dy}{dx} = -3$

$-3 = 3(1) + C$

$C = -6$

$\dfrac{dy}{dx} = 3x - 6$

$y = \dfrac{3x^2}{2} - 6x + C$

when $x = 1$ $y = 0$

$0 = \dfrac{3(1)}{2} - 6(1) + C$

$C = \dfrac{9}{2}$

$y = \dfrac{3x^2}{2} - 6x - \dfrac{9}{2}$

Done poorly

This style of question is common.

Exam Mathematics Trial 2015	MATHEMATICS Suggested Solutions	: Question...... 13 e f	Marker's Comments

e i $A_1 = M \times 1.01 - 500$

This was done very well

ii. $A_2 = A_1 \times 1.01 - 500$

$= M \times 1.01^2 - 500 \times 1.01 - 500$

$= M \times 1.01^2 - 500(1 + 1.01)$

part ii

make sure you 'show' these equations

$A_3 = A_2 \times 1.01 - 500$

$= M \times 1.01^3 - 500(1 + 1.01 + 1.01^2)$

iii. $A_n = M \times 1.01^n - 500(1 + 1.01 + 1.01^2 + \ldots + 1.01^{n-1})$

iv. $n = 5 \times 12$

$= 60$

some errors included:

$n = 6$ or $n = 120$

$A_{60} = 0$

$0 = M \times 1.01^{60} - 500(1 + 1.01 + 1.01^2 + \ldots + 1.01^{59})$

$500\left(\frac{1.01^{60} - 1}{0.01}\right) = M \times 1.01^{60}$

$M = \frac{500(1.01^{60} - 1)}{1.01^{60} \times 0.01} = \$22477.5192$

$= \$22500$ (nearest \$100)

f. i. $\alpha + \beta = \frac{-b}{a} = \frac{5}{1} = 5$

$\alpha\beta = \frac{c}{a} = 2$

ii. $\left(\alpha^2 + \frac{1}{\beta}\right)\left(\beta^2 + \frac{1}{\alpha}\right)$

Easier method: expand the brackets

Expand

$\alpha^2\beta^2 + \alpha + \beta + \frac{1}{\alpha\beta}$

$= (2)^2 + 5 + \frac{1}{2}$

$= 9\frac{1}{2}$

Exam 2U Maths	MATHEMATICS	: Question 14	Q14 Generally well answered
	Suggested Solutions		Marker's Comments

a) $\sum_{n=1}^{3} n^2(n+1) = 1\times 2 + 4\times 3 + 9\times 4$
$= \underline{\underline{50}}$

Well answered

b) $\log_2(mn)^3 = 3\log_2(mn)$
$= 3(\log_2 m + \log_2 n)$
$= 3(0\cdot2134 + 0\cdot3142)$
$= 1\cdot5828$
$= \underline{\underline{1\cdot58}}$ (to 2 dp)

Learn the log laws.

Remember to round off to required accuracy.

c) i) $y = -x^3 + 3x^2 + 9x - 11$
$\frac{dy}{dx} = -3(x^2 - 2x - 3)$
$= \underline{\underline{-3(x-3)(x+1)}}$

This is a 'Show that' question. As there are not many steps for 2 marks, it is important not to miss the 2nd line out. The 3rd line is given to you!

(ii) Solve $\frac{dy}{dx} = 0$, $x = 3, -1$

$x = 3,\ y = 16$ $\quad\therefore$ $\underline{(3,16)\ (-1,-16)\text{ are stationary points}}$
$x = -1,\ y = -16$

$\frac{d^2y}{dx^2} = -6x + 6$
$x = 3,\ \frac{d^2y}{dx^2} = -18 + 6 = -12 < 0$
$\therefore$ $\underline{(3,16)\text{ is a local maxima}}$
$x = -1,\ \frac{d^2y}{dx^2} = 6 + 6 = 12 > 0$
$\therefore$ $\underline{(-1,-16)\text{ local minima}}$

From here is a 'Show that'

You need to put both the value of -12 for $\frac{d^2y}{dx^2}$ and the sign < 0.

Many students subbed $x = -1$ in incorrectly.

(or alternatively

x	-2	-1	0	2	3	4
$\frac{dy}{dx}$	-15	0	9	9	0	-15
	$-$	0	$+$	$+$	0	$-$

)

Again put values and signs and put $x, \frac{dy}{dx}$ in. Some students put in x, y by mistake.

Exam 2U Maths	MATHEMATICS : Question 14	
	Suggested Solutions	Marker's Comments

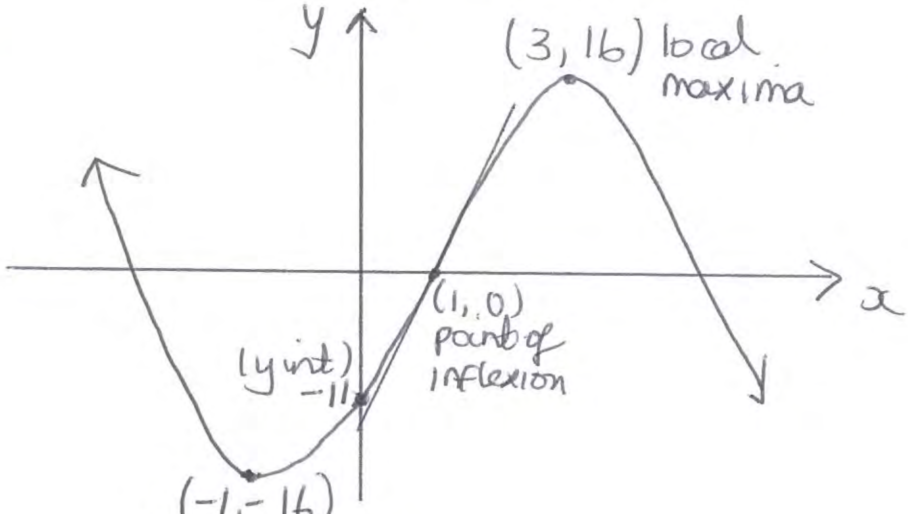
C(iii) $\frac{d^2y}{dx^2} = 0$, $x = 1$, $y = 0$ $\underline{(1, 0)}$

x	0	1	2
$\frac{d^2y}{dx^2}$	6 >0	0	-6 <0

(Change in sign of $\frac{d^2y}{dx^2}$, $\therefore$ change in concavity, $\therefore (1, 0)$ point of inflexion.)

Question said 'find' and was worth 1 mark. On this occasion the table was not deemed necessary. I have put this in for other occasions.

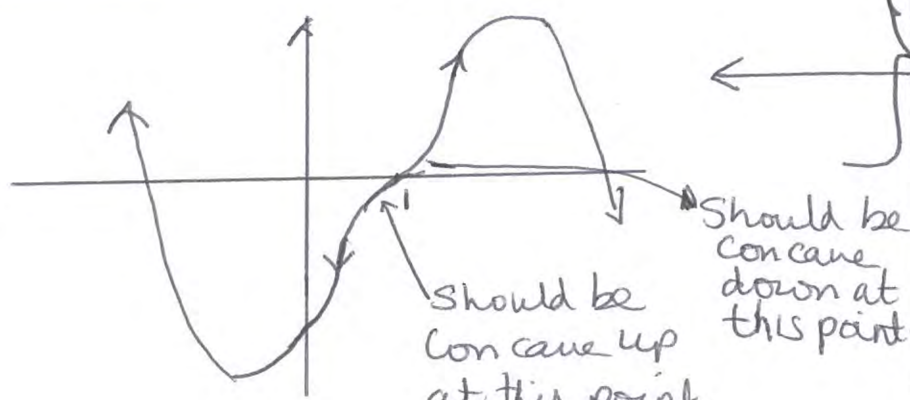
(iv)



Many points need to be noted for (iv)

- *Use one single clear line, not several sketchy lines*
- *the point (3, 16) needs to be the top point of the curve locally – not to the side. Similar for minima at bottom.*
- *Don't waste time on detailed numbered axes measured unit by unit.*
- *Many students showed point of inflexion incorrectly like this.*

<u>Incorrect</u> showing of point of inflexion



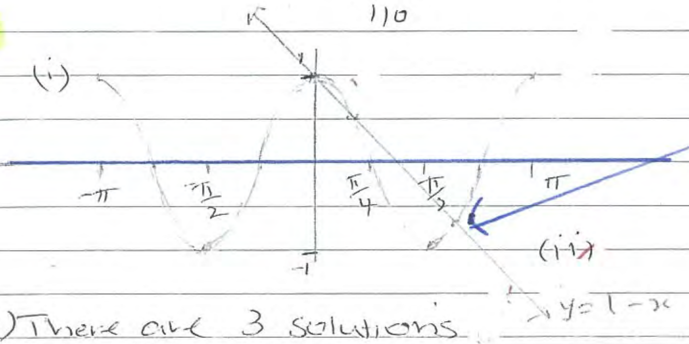
(v) Concave up $\frac{d^2y}{dx^2} > 0$

$$-6x + 6 > 0$$
$$-6x > -6$$
$$\underline{x < 1}$$

Exam 2U Maths	MATHEMATICS	: Question 14

Suggested Solutions	Marker's Comments
d) i) <u>In ΔABC, ΔHBA</u> $\angle B$ is common $\angle HAB = \angle ACB = \alpha$ (given) $\therefore$ <u>$\Delta ABC \parallel\!\mid \Delta HBA$</u> (equiangular) (ii) $\dfrac{BC}{AB} = \dfrac{AB}{HB}$ $\dfrac{BC}{6} = \dfrac{6}{4}$ $BC = \dfrac{36}{4} = \underline{\underline{9 \text{ units}}}$.	• No marks for top line this time but needs to be in to cover other marking schemes. • You do not need to show the 3rd pair of angles equal for equiangular proof.

Exam MATHEMATICS : Question 15

Suggested Solutions	Marker's Comments
a) i) $\int_1^6 \frac{1}{x+2}\,dx = \left[\ln(x+2)\right]_1^6 = \ln 8 - \ln 4 = \ln 2$	Common mistake $\ln 8 - \ln 2 = \ln 4$
ii) $\int_0^1 (e^{2x}+1)\,dx = \left[\frac{e^{2x}}{2}+x\right]_0^1 = \frac{e^2}{2}+1-\frac{1}{2} = \frac{e^2}{2}+\frac{1}{2}$	$\frac{e^2}{2}-\frac{1}{2}$
iii) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 3\sec^2 x\,dx = \left[3\tan x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = 3\tan\frac{\pi}{3} - 3\tan\frac{\pi}{4}$	
$= 3\left(\frac{\sqrt{3}}{1}\right) - 3(1) = 3\sqrt{3} - 3$	
b) i) $P(WW) = \frac{n}{100} \times \frac{n-1}{99} = \frac{2}{275}$	
$\frac{n^2-n}{9900} = \frac{2}{275}$	
$n^2 - n = 72$	
$n^2 - n - 72 = 0$	
$(n-9)(n+8) = 0$	
$n = 9,\ n = -8$ (no solution)	
$\therefore$ 9 tickets were bought	
ii) $P(\text{at least one}) = 1 - P(\text{none})$	
$= 1 - \left(\frac{91}{100} \times \frac{90}{99}\right)$	Common mistake $\frac{90}{100}$
$= 1 - \frac{91}{110}$	
$= \frac{19}{110}$	
c) (i) 	Drawn inaccurately
(iii) There are 3 solutions	

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(d) 3 values : $x = 0$, $x = \frac{\pi}{4}$, $x = \frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} \cos^2 x \doteq \frac{h}{2}\left[f(a) + 2f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Some students used Simpson rule

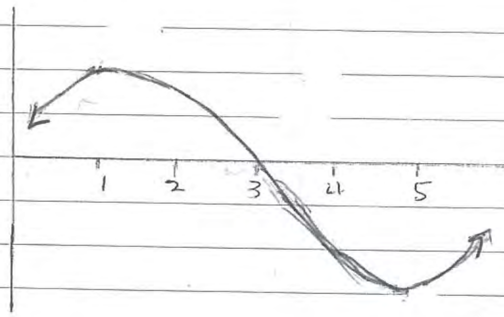
$$\doteq \frac{\frac{\pi}{2} - 0}{2}\left[f(0) + 2f\left(\frac{\pi}{4}\right) + f\left(\frac{\pi}{2}\right)\right]$$

$$\doteq \frac{\pi}{8}\left[1 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 0\right]$$

$$\doteq 0.785 \text{ units}^2$$

Carelessness used exact values instead

e)



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Suggested Solutions	Marker's Comments

(ii) $A = x \times x + 4xy$

$= x^2 + 4x \times \frac{32}{x^2}$

$= x^2 + \frac{128}{x}$

Some students forgot that it was an open box from the top.

(iii) for area to be minimum, $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = 2x - \frac{128}{x^2}$

$\frac{dA}{dx} = 0 \Rightarrow 2x = \frac{128}{x^2}$

$\Rightarrow x^3 = 64$

$x = 4$

minimum requirement: $\frac{d^2A}{dx^2} = 2 + \frac{128 \times 2}{x^3} > 0$ for $x = 4$

deducted ½ mark for students who did not show that area is minimum for $x = 4$.

4, 4, 2 are the dimensions of the box.

f) $V = \pi \int_1^2 x^2 \, dy$

$y = \frac{2}{1+x^2}$

$y(1+x^2) = 2$

$y + yx^2 = 2$

$yx^2 = 2 - y$

$x^2 = \frac{2-y}{y}$

$= \frac{2}{y} - 1$

$= \pi \int_1^2 \left(\frac{2}{y} - 1\right) dy$

$= \pi \left(2\ln y - y\right)_1^2$

$= \pi \left(2\ln 2 - 2 - 2\ln 1 + 1\right)$

$= \pi \left(2\ln 2 - 1\right)$ as $\ln 1 = 0$

Some students forgot to make x as the subject.

END